Pre-Calculus 12 Sample Examination Questions – Solutions

Note Title 1/11/2016

Selected Response:

1) C	7) B	13) D	19) C	25) A
2) C	8) B	14) C	20) C	26) B
3) D	9) C	15) B	21) A	27) C
4) D	10) B	16) D	22) B	28) C
5) D	11) A	17) B	23) B	29) B
6) C	12) B	18) A	24) A	30) B

Constructed Response

31a) $y = k(x+4)(x+1)(x-3)^2$

$(2,9)$: $9 = k(2+4)(2+1)(2-3)^2$

$9 = k(6)(3)(-1)^2$

$9 = 18k$

$\frac{1}{2} = k$

$\therefore y = \frac{1}{2}(x+4)(x+1)(x-3)^2$

31b) reflection in y-axis
h. trans of +1
v. stretch of 3

$\left.\right\}$ $(x,y) \to (-x-1, 3y)$

$y = 3\sqrt{-(x-1)}$

Check: $x=1$ $y = 3\sqrt{0}$ ✓

$x=0$ $y = 3\sqrt{1} = 3$ ✓

31c) h. trans: + 4

doubles each time

$y = 2^x + 4$

Check: $x = 3$ $y = 2^3 + 4 = 12$ ✓

31d) $y = \dfrac{(x+1)(x-1)}{(x-1)(x+3)(x-3)}$

31e) s. axis: $y=1$
amp: 3
period: π
h. trans: $\pi/3$

$y = 3\sin 6(x - \pi/3) + 1$

32a) $\underbrace{\sin^2(2x)}+\cos(x)=0$

$2\sin x\cos x + \cos x = 0$

$\cos x\,[2\sin x + 1] = 0$

$\cos x = 0$	$2\sin x + 1 = 0$
$x = \frac{\pi}{2}+\pi k;$	$\sin x = -\frac{1}{2}$
$k\in\mathbb{Z}$	$x = -\frac{\pi}{6}+2\pi k;\ k\in\mathbb{Z}$
	$x = \frac{7\pi}{6}+2\pi k;\ k\in\mathbb{Z}$

32b) $2\sin^2x + \cos x - 1 = 0$

$2[1-\cos^2x]+\cos x - 1 = 0$

$2-2\cos^2x+\cos x - 1 = 0$

$-2\cos^2x+\cos x+1=0$

$2\cos^2x-\cos x-1=0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$	$\cos x - 1 = 0$
$\cos x = -\frac{1}{2}$	$\cos x = 1$
$x = \frac{2\pi}{3}+2\pi k;$	$x = 0 + 2\pi k;\ k\in\mathbb{Z}$
$x = \frac{4\pi}{3}+2\pi k;$	
$k\in\mathbb{Z}$	

32c) $2\cos(2x)\cos(\frac{\pi}{5})+2\sin(2x)\sin(\frac{\pi}{5})=1$

$\underbrace{\cos(2x)\cos(\frac{\pi}{5})+\sin(2x)\sin(\frac{\pi}{5})}=\frac{1}{2}$

$\cos(2x-\frac{\pi}{5}) = \frac{1}{2}$

let $\theta = 2x-\frac{\pi}{5}$

$\therefore \cos\theta = \frac{1}{2}$

$\theta_1 = \frac{\pi}{3}+2\pi k$	$\theta_2 = \frac{5\pi}{3}+2\pi k$
$2x-\frac{\pi}{5} = \frac{\pi}{3}+2\pi k$	$2x-\frac{\pi}{5} = \frac{5\pi}{3}+2\pi k$
$2x = (\frac{\pi}{3}+\frac{\pi}{5})+2\pi k$	$2x = (\frac{5\pi}{3}+\frac{\pi}{5})+2\pi k$
$2x = (\frac{5\pi}{15}+\frac{3\pi}{15})+2\pi k$	$2x = (\frac{25\pi}{15}+\frac{3\pi}{15})+2\pi k$
$2x = \frac{8\pi}{15}+2\pi k$	$2x = \frac{28\pi}{15}+2\pi k$
$x = \frac{4\pi}{15}+\pi k,\ k\in\mathbb{Z}$	$x = \frac{14\pi}{15}+\pi k,$

$k\in\mathbb{Z}$

32d) $\frac{2!\,n!}{4!\,(n-5)!} = \frac{(n-1)!}{(n-4)!}$

$\frac{\cancel{2!}\,n(n-1)(n-2)(n-3)(n-4)\cancel{(n-5)!}}{4\cdot3\cdot\cancel{2!}\,\cancel{(n-5)!}} = \frac{(n-1)(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}}$

$\frac{n(n-1)(n-2)(n-3)(n-4)}{4\cdot 3} = (n-1)(n-2)(n-3)$

$\frac{n(n-4)}{12} = 1$ iff $n\neq 1, n\neq 2$ and $n\neq 3$

$n(n-4) = 12$

$n^2-4n-12=0 \longrightarrow (n-6)(n+2)=0$

$n-6=0 \qquad n+2=0$

$\boxed{n=6} \qquad \cancel{n=-2}$

not possible in context of factorial

32e) $\frac{x!}{(x-2)!} = 20$

$\frac{(x)(x-1)(x-2)!}{(x-2)!} = 20$

$(x)(x-1) = 20$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x-5=0 \quad x+4=0$

$\boxed{x=5} \quad \cancel{x=-4}$

32g) $\ln(x+1) + \ln(x-2) = \ln(4)$

$\ln[(x+1)(x-2)] = \ln 4$

$(x+1)(x-2) = 4$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x-3=0 \quad x+2=0$

$\boxed{x=3} \quad \cancel{x=-2}$

Check: $\ln(4) + \ln(1) = \ln(4)$ ✓

32j) $3^{x+2} - 4 = 6$

$3^{x+2} = 10$

$\log_3(10) = x+2$

$\frac{\log 10}{\log 3} = x+2$

$x = -2 + \frac{\log 10}{\log 3}$

$x \doteq 0.0959$

32f) $\log_2(5x-2) - \log_2 2 = \frac{1}{2}\log_2 36 + 2\log_2 3$

$\log_2 \frac{(5x-2)}{2} = \log_2(\sqrt{36})(3^2)$

$\log_2\left(\frac{5x-2}{2}\right) = \log_2(54)$

$\therefore \frac{5x-2}{2} = 54$

$5x - 2 = 108$

$5x = 110$

$x = 22$

Check: $\log_2(108) - \log_2 2 = \log_2 6 + \log_2 9$

$\log_2(54) = \log_2(54)$ ✓

32h) $\frac{1}{3}\log_3 27 + \log_3 x = 4\frac{1}{2}$

$3 + \log_3 x = 2$

$\log_3 x = -1$

$3^{-1} = x$; $\boxed{x = \frac{1}{3}}$

32i) $3\log_4(2x+1) - \log_4(x-2) = 1$

$\log_4 \frac{(2x+1)^3}{x-2} = 1$

$\frac{(2x+1)^3}{x-2} = 4^1$

$(2x+1)^3 = 4(x+2)$

$8x^3 + 3(4x^2) + 3(2x) + 1 = 4x + 8$

$\underbrace{8x^3 + 12x^2 + 6x - 7 = 0}$

$x = \frac{1}{2}$ is only possible solution.

It must be rejected since $\log_4(x-2)$ would yield ← negative argument

0 real solutions

32 k) $4(2^x)^3 + 2(2^x) - 18 = 40(2^x)^2 + 2$

let $2^x = k$

$4k^3 - 40k^2 + 2k - 20 = 0$

$4k^2(k-10) + 2(k-10) = 0$

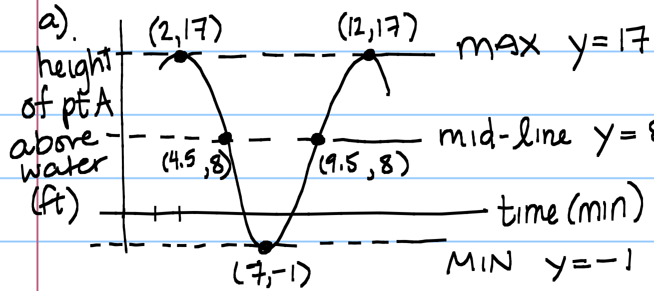
$(k-10)(4k^2+2) = 0$

$k - 10 = 0 \qquad 4k^2 + 2 = 0$

$\boxed{k = 10} \qquad$ ↑ No real solution.

33) Waterwheel Question

a)



b) Eqn of midline: $y = 8$

Amplitude: 9 feet

Period: 10 seconds

(6 revolutions in 60 seconds)

c) $y = 9\cos\left[\frac{2\pi}{10}(x-2)\right] + 8$

d) $x = 135$: $y = 9\cos\left[\pi/5\,(135-2)\right] + 8$

$y = 5.22$ ft

e) $y = 10$: $10 = 9\cos\left[\pi/5\,(x-2)\right] + 8$

$2 = 9\cos\theta$, where $\theta = \pi/5\,(x-2)$

$\frac{2}{9} = \cos\theta$

$\theta_1 \doteq 1.35 + 2\pi k$

$\pi/5\,(x-2) \doteq 1.35 + 2\pi k$

$x - 2 \doteq 2.15 + 10k$

$x \doteq 4.15 + 10k, k \in \mathbb{Z}$

$\theta_2 = 2\pi - \theta_1$

$\theta_2 \doteq 4.94 + 2\pi k$

$\pi/5\,(x-2) = 4.94 + 2\pi k$

$x - 2 = 7.86 + 10k, k \in \mathbb{Z}$

$x = 9.86 + 10k; k \in \mathbb{Z}$

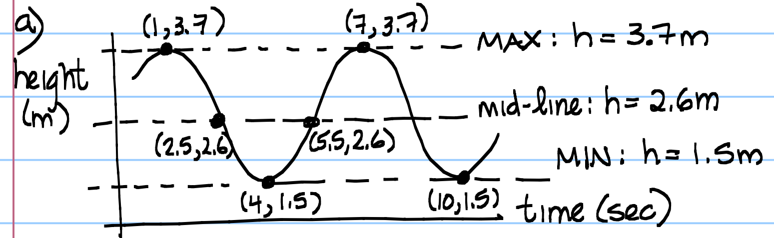
∴ Reaches a height of 10 m at $t = 4.15$ sec

$t = 9.86$ sec and

$t = 14.15$ sec

Oil Jack Question

a)



b) $h = 1.1\cos\left(\frac{2\pi}{6}(t-1)\right) + 2.6$ or $h = 1.1\sin\left(\frac{2\pi}{6}(t-5.5)\right) + 2.6$

c) i) $t = 5.5$: $h = 1.1\cos\left(\frac{\pi}{3}(5.5-1)\right) + 2.6$

$h = 2.6$ m

ii) $t = 9.3$: $h = 1.1\cos\left(\pi/3(9.3-1)\right) + 2.6$

$h = 1.78$ m

d) $h = 2$: $2 = 1.1\cos\left(\pi/3(t-1)\right) + 2.6$

$-0.6 = 1.1\cos\theta$ where $\theta = \pi/3(t-1)$

$-0.5\overline{45} = \cos\theta$

$\theta_1 = 2.15 + 2\pi k$ | $\theta_2 = 2\pi - \theta_1 = 4.135 + 2\pi k$

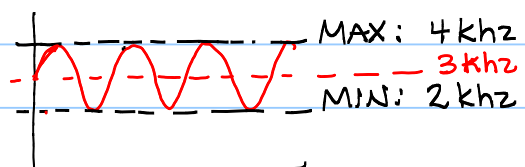
$\pi/3(t-1) = 2.15 + 2\pi k$ | $\pi/3(t-1) = 4.135 + 2\pi k$

$t-1 = 2.05 + 6k$ | $t-1 = 3.95 + 6k$

$t = 3.05 + 6k, k\in\mathbb{Z}$ | $t = 4.95 + 6k, k\in\mathbb{Z}$

$\therefore$ Will reach height at $t = 3.05$ sec $t = 4.95$ sec and every 6 seconds after each of these times.

Car Alarm Question



Period of original, $\frac{1}{6}(y-1) = \sin 90x$, is $\frac{2\pi}{90} = \frac{\pi}{45}$ or 0.07 sec

Since new frequency is twice as fast the period would be $\frac{0.07}{2}$ sec or 0.035 sec

amplitude: 1

s. axis: $y = 3$

period: 0.035 sec

$y = \sin(180x) + 3$

or

$y - 3 = \sin(180x)$

34) Earth Population Question

a) $P = 7.39(1.023)^t$

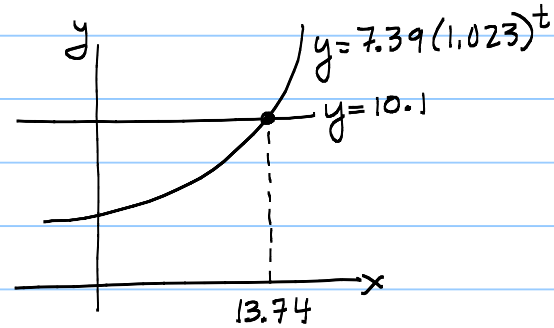
b) $P = 10.1$: $10.1 = 7.39(1.023)^t$

$1.3667 = 1.023^t$

$\log 1.3667 = t \log 1.023$

$\frac{\log 1.3667}{\log 1.023} = t$

$t = 13.74$ years



∴ Earth's population should reach 10.1 billion in approx 13.7 years

District's Population Question

a) $A_0 = 3500$: $A = 3500(1 + 0.005)^{96}$

$i = \frac{0.06}{12}$ $A = \$5649.50$

$n = 8(12)$

b) $A = 2A_0$: $2A_0 = A_0(1 + 0.005)^{12x}$

$i = \frac{0.06}{12}$ $2 = (1.005)^{12x}$

$n = 12(x)$ $\log_{1.005} 2 = 12x$

x is # of years

$\frac{\log 2}{\log 1.005} = 12x$

$138.976 = 12x$

$x = 11.58$ years

∴ It will take approx 11 years and 7 months for an investment to double

35) #1 $x^2 - 4 = \frac{x^2 - 4}{x - 4}$

$(x^2 - 4)(x - 4) = x^2 - 4$

$x^3 - 4x^2 - 4x + 16 = x^2 - 4$

$x^3 - 5x^2 - 4x + 20 = 0$

$x^2(x - 5) - 4(x - 5) = 0$

$(x^2 - 4)(x - 5) = 0$

$x^2 - 4 = 0$; $x - 5 = 0$

$x^2 = 4$ $\boxed{x = 5}$

$\boxed{x = \pm 2}$

∴ Points of intersection are (2,0); (-2,0) and (5,21)

filled into $y = x^2 - 4$ or $y = \frac{x^2 - 4}{x - 4}$

35) #2 $\sqrt{5x^2-20} = 4 - x^2$

$5x^2 - 20 = (4-x^2)^2$

$5x^2 - 20 = 16 - 8x^2 + x^4$

$0 = x^4 - 13x^2 + 36$

$0 = (x^2-9)(x^2-4)$

$x^2 - 9 = 0 \quad ; \quad x^2 - 4 = 0$

$x^2 = 9 \quad ; \quad x^2 = 4$

$\underbrace{x = \pm 3} \quad ; \quad \boxed{x = \pm 2}$

extraneous
since $f(\pm 3) = 25$
$g(\pm 3) = -5$

36 a) $\cot^2 x - \cos^2 x$

$\dfrac{\cos^2 x}{\sin^2 x} - \cos^2 x$

$\dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\cos^2 x \sin^2 x}{\sin^2 x}$

$\dfrac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$

$\dfrac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x}$

$\dfrac{\cos^2 x}{\sin^2 x} \cdot \cos^2 x$

$\cot^2 x \cos^2 x$ // QED

b) $\sin 3x = \sin(2x + x)$

$\underbrace{\sin 2x} \cos x + \underbrace{\cos 2x} \sin x$

$2\sin x \cos x \cdot \cos x + \underbrace{(1 - 2\sin^2 x)\sin x}$

$2\sin x \underbrace{\cos^2 x} + \sin x - 2\sin^3 x$

$2\sin x (1 - \sin^2 x) + \sin x - 2\sin^3 x$

$2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$3\sin x - 4\sin^3 x$ // QED

c) $\dfrac{\sin^3 x}{\cos x} + \sin x \cos x$

$\dfrac{\sin^3 x}{\cos x} + \dfrac{\sin x \cos x \cdot \cos x}{\cos x}$

$\dfrac{\sin^3 x + \sin x \cos^2 x}{\cos x}$

$\dfrac{\sin x (\sin^2 x + \cos^2 x)}{\cos x}$

$\dfrac{\sin x}{\cos x} \cdot 1 = \tan x$ // QED

36d) $\frac{1}{\cos\theta} - \cos\theta$

$\frac{1-\cos^2\theta}{\cos\theta}$

$\frac{\sin^2\theta}{\cos\theta}$

$\sin^2\theta \cdot \frac{1}{\cos\theta}$

$\sin^2\theta \sec\theta$ // QED

e) $\cos^4 x - \sin^4 x$

$\underbrace{(\cos^2 x - \sin^2 x)}_{\cos(2x)} \cdot \underbrace{(\cos^2 x + \sin^2 x)}_{1}$

$\cos(2x)$ // QED

f) $\frac{\sin 2x}{1+\cos 2x}$

$\frac{2\sin x\cos x}{1+[2\cos^2 x - 1]} = \frac{2\sin x \cos x}{2\cos^2 x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$ // QED

g) $\csc x - \cos x \csc x$

$\csc x\,(1-\cos x)\cdot\frac{(1+\cos x)}{(1+\cos x)}$

$\frac{\csc x\,(1-\cos^2 x)}{1+\cos x}$

$\frac{\csc x \cdot \sin^2 x}{1+\cos x}$

$\frac{\frac{1}{\sin x}\cdot \sin^2 x}{1+\cos x}$

$\frac{\sin x}{1+\cos x}$ // QED

h) $\frac{\cos\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$

$\frac{(\cos\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta} = \frac{1+\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$ // QED

37) #1 $2x^3+5x^2-5x+1=0$

Rational Roots:

$\{\pm\frac{1}{2}, 1\}$

$f(1) = 2+5-5+1 \neq 0$

$f(-1) = -2+5+5+1 \neq 0$

$f(\frac{1}{2}) = \frac{2}{8}+\frac{5}{4}-\frac{5}{2}+1 = 0$ ✓

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 5 & -5 & 1 \\ & & 1 & 3 & -1 \\ \hline & 2 & 6 & -2 & 0 \end{array}$$

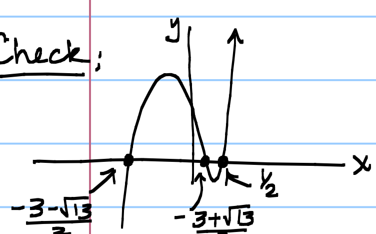
$\therefore\ 2x^3+5x^2-5x+1 = (x-\frac{1}{2})(2x^2+6x-2)$

$f(x)=0$ when

$x-\frac{1}{2}=0$ and $2x^2+6x-2=0$ or $x^2+3x-1=0$

$\boxed{x=\frac{1}{2}}$ $\quad x = \frac{-3\pm\sqrt{9-4(1)(-1)}}{2(1)} = \boxed{\frac{-3\pm\sqrt{13}}{2} = x}$

Check:



37) #2 $f(x) = g(x)$

$$x^3 - 3x^2 = 4x - 12$$
$$\underbrace{x^3 - 3x^2} - 4x + 12 = 0$$
$$x^2(x-3) - 4(x-3) = 0$$
$$(x-3)(x^2-4) = 0$$
$$x - 3 = 0 \quad \text{and} \quad x^2 - 4 = 0$$
$$\boxed{x = 3} \qquad x^2 = 4$$
$$\boxed{x = \pm 2}$$

Intersection Points:

$(3, 0)$; $(2, -4)$; $(-2, -20)$

$\uparrow g(3)$ or $f(3)$; $\uparrow g(+2)$ or $f(2)$; $\uparrow g(-2)$ or $f(-2)$

#4 $3\sec^2 x = 12$

$$\sec^2 x = 4$$
$$\sec x = \pm 2$$
$$\cos x = \pm \frac{1}{2}$$
$$\therefore \boxed{x = \pi/3 ;\ 2\pi/3 ;\ 4\pi/3 ,\ 5\pi/3}$$

38) #1 $\log_2(4x) - 1$

$$\underbrace{\log_2 4} + \log_2 x - 1$$
$$2 + \log_2 x - 1$$
$$\underbrace{1} + \log_2 x$$
$$\log_2 2 + \log_2 x$$
$$\log_2 (2 \cdot x)$$
$$\log_2 (2x), \text{ QED}$$

#3 $3\sin(2x) + 5 = 7$

$3\sin(\theta) = 2$ where $\theta = 2x$

$$\sin\theta = \frac{2}{3}$$

$\theta_1 = 41.8^\circ + 360^\circ k$	$\theta_2 = (180^\circ - \theta_1) + 360^\circ k$
$2x = 41.8^\circ + 360^\circ k$	$2x = 138.2^\circ + 360^\circ k$
$x = 20.9^\circ + 180^\circ k;\ k \in \mathbb{Z}$	$x = 69.1^\circ + 180^\circ k$ $k \in \mathbb{Z}$

$\therefore$ Between -180° and 360°

$$\boxed{\begin{array}{l} x = 20.9^\circ, 200.9^\circ, -159.1^\circ \\ x = 69.1^\circ, 249.1^\circ, -110.9^\circ \end{array}}$$

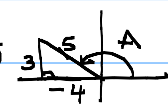
#5 $\sin^2 x = 3\sin x - 2$

$$\sin^2 x - 3\sin x + 2 = 0$$

let $p = \sin x$

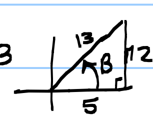
$$p^2 - 3p + 2 = 0$$
$$(p-2)(p-1) = 0$$
$$p - 2 = 0 \qquad p - 1 = 0$$
$$p = 2 \qquad p = 1$$
$$\cancel{\sin x = 2} \qquad \sin x = 1$$
$$\boxed{x = \pi/2 ,\ 5\pi/2}$$

38) #2 $\sin A = 3/5$



$\therefore \cos A = {-4}/5$ and $\tan A = {-3}/4$

$\sin B = 12/13$



$\therefore \cos B = 5/13$ and $\tan B = \frac{12}{5}$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{-3/4 - 12/5}{1 + (-3/4)(12/5)} = \frac{\frac{-15}{20} - \frac{48}{20}}{1 - \frac{36}{20}} = \frac{-\frac{63}{20}}{\frac{-16}{20}} = \frac{63}{16}$$

38) #3 $\log(a), \log(ab), \log(ab^2), \log(ab^3)$

$$\underset{t_1}{\log(a)},\ \underset{t_2}{\log(ab)},\ \underset{t_3}{\log(ab^2)},\ \underset{t_4}{\log(ab^3)}$$

$t_1 = \log(a)$

$t_2 = \log(a) + \log(b)$

$t_3 = \log(a) + \log(b^2) = \log(a) + 2\log(b)$

$t_4 = \log(a) + \log(b^3) = \log(a) + 3\log(b)$

$$\therefore \left.\begin{aligned} t_2 - t_1 &= \log(b) \\ t_3 - t_2 &= \log(b) \\ t_4 - t_3 &= \log(b) \end{aligned}\right\}$$

∴ These terms have a common difference of $\log(b)$ and the sequence is arithmetic.

#4 $\sqrt{2^x} - \dfrac{12}{\sqrt{2^x}} = 1$

let $k = \sqrt{2^x}$

$\therefore k - \dfrac{12}{k} = 1$

$\dfrac{k^2}{k} - \dfrac{12}{k} = 1$

$\dfrac{k^2 - 12}{k} = 1$

$k^2 - 12 = k$

$k^2 - k - 12 = 0$

$(k-4)(k+3) = 0$

$k - 4 = 0 \qquad k + 3 = 0$

$k = 4 \qquad k = -3$

$\sqrt{2^x} = 4 \qquad \cancel{\sqrt{2^x} = -3}$

Not possible

$2^x = 16$

$2^4 = 16$

$\therefore x = 4$

#5 $\log_2(2x-6) = \log_4 x$

$\dfrac{\log(2x-6)}{\log 2} = \dfrac{\log x}{\log 4}$

since $4 = 2^2$ $\log 4 = \log 2^2$

$\therefore \log 4 = 2\log 2$

Thus: $\dfrac{\log(2x-6)}{\log 2} = \dfrac{\log x}{2\log 2}$

$\log(2x-6) = \frac{1}{2}\log x$

$\log(2x-6) = \log x^{1/2}$

$2x - 6 = \sqrt{x}$

$(2x-6)^2 = x$

$4x^2 - 24x + 36 = x$

$4x^2 - 25x + 36 = 0$

$x = \dfrac{25 \pm \sqrt{25^2 - 4(4)(36)}}{2(4)} = \dfrac{25 \pm \sqrt{49}}{8}$

$x = \dfrac{25 \pm 7}{8}$; $x = 9/4$; $\boxed{x = 4}$

doesn't work since it creates a negative argument.

38) #6 $f(g(x)) = 9x^2 - 6x + 5$

$f(\underbrace{1-3x}) = 9x^2 - 6x + 5$

we need to determine $f(1)$

∴ We shoud determine for what value of x

$1-3x = 1$

$1 - 3x = 1$

$-3x = 0$

$x = 0$

thus $f(1-3x) = 9x^2 - 6x + 5$

when $x=0$: $f(1-3(0)) = 9(0)^2 - 6(0) + 5$

$f(1) = 5$ //

#7 $P(x) = 3x^3 - 6ax^2 - 4ax + 8a$

Since root is $2a$

$$\begin{array}{r|rrrr} 2a & 3 & -6a & -4a & 8a \\ & & 6a & 0 & -8a \\ \hline & 3 & 0 & -4a & 0 \end{array}$$

$\therefore P(x) = (x-2a)(3x^2 - 4a)$

$P(x) = 0$ when

$x - 2a = 0$ and $3x^2 - 4a = 0$

$x = 2a$ $\quad$ $3x^2 = 4a$

$x^2 = \frac{4}{3}a$

$x = \pm \frac{2}{\sqrt{3}}\sqrt{a}$

$\boxed{x = \pm \frac{2\sqrt{3a}}{3}}$

OR

$P(x) = \underbrace{3x^3 - 6ax^2} - \underbrace{4ax + 8a}$

$= 3x^2(x-2a) - 4a(x-2a)$

$P(x) = (x-2a)(3x^2 - 4a)$

↓ etc

#8 $\frac{e^{-x}}{e^{-x}+1} + \frac{e^{x}}{e^{x}+1}$

$= \frac{e^{-x}(e^x+1) + e^x(e^{-x}+1)}{(e^{-x}+1)(e^x+1)}$

$= \frac{e^0 + e^{-x} + e^0 + e^x}{e^0 + e^{-x} + e^x + 1}$

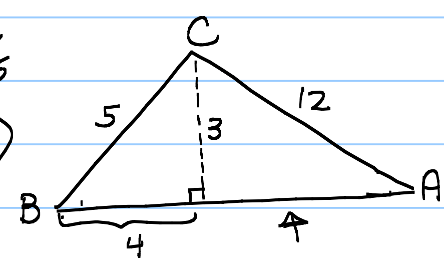
$= \frac{2 + e^{-x} + e^x}{2 + e^{-x} + e^x} = 1$ // QED

#9 $\sin B = 3/5$

$\sin C = 1/4$

or

$\sin C = 3/12$



∴ AB : AC

$3\sqrt{15} : 12$

or $\sqrt{15} : 4$

$12^2 = 3^2 + x^2$

$144 = 9 + x^2$

$135 = x^2$

$x = \sqrt{135}$

$x = 3\sqrt{15}$

#10 $50e^{0.1t} = 500 - 450e^{-0.1t}$

let $e^{0.1t} = p$

$50p = 500 - 450p^{-1}$

$50p - 500 = -\frac{450}{p}$

$50p^2 - 500p = -450$

$50p^2 - 500p + 450 = 0$

$p^2 - 10p + 9 = 0$

$(p-9)(p-1) = 0$

$p - 9 = 0$ $\quad$ $p + 1 = 0$

$p = 9$ $\quad$ $p = -1$ ← Not possible

$e^{0.1t} = 9$

$\ln 9 = 0.1t$

$t = 10\ln 9$

$\boxed{t \doteq 21.97 \text{ days}}$

∴ Populations are equal after approx 22 days